Journal of Studies in Social Sciences and Humanities
http://www.jssshonline.com/
Volume 7, No. 4, 2021, 433-450
ISSN: 2413-9270

Improving Teachers' Pedagogical Knowledge of Teaching Mathematics: Metacognitive Skills and Strategies Application

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Abstract

The study aims to ascertain the actions of the rural school teachers' application of metacognitive skills and strategies in their teaching, and how it supported or improved, their pedagogical knowledge of teaching mathematics. The researcher developed the discussion by reporting and referring to the research that was conducted amongst forty-eight participants (N=48 teachers: males=21 and females=27) through cluster sampling technique. Explanatory sequential mixed-method design was used. These participants were rural mathematics teachers in South Africa. Questionnaires were used for data collection. Classroom observations and interviews were used for additional data collection. A statistical package (R software) for quantitative and content analysis for qualitative were used for data analysis. This research showed that teachers could actively foster metacognition in a variety of ways to improve their pedagogical knowledge. However, a noteworthy point of interest in this study was that, prior to this, the teachers claimed that they applied metacognition, yet teachers' results year after year had not been improving. Does the fault lie in their training, the application level, or accountability level? The study revealed that there was a misunderstanding beforehand; the participants, in fact, did not understand the concept (metacognition) or application thereof in their teaching practices that is why they indicated that they applied metacognition as evidenced in their qualitative response. Recommendations are made for the above findings.

Key words: Metacognition, pedagogical knowledge, rural schools, skills and strategies, teachers.

Introduction

The teaching of mathematics is not only a concern in South African schools but in other parts of the world where strategies need to be put in place to assist learners in formulating, using, and interpreting mathematics in various contexts. In order to describe, explain and predict phenomena to assist in making constructive and reflective decisions (Suciati & Febriyanti, 2020), focus is placed on mathematical reasoning, using relevant mathematical concepts, procedures, facts, and tools (Ojose, 2011; Stacey & Turner, 2015; Suciati & Febriyanti, 2020). The National Centre for Educational Statistics indicates that in the USA only half of senior secondary school graduates enrol for mathematics courses beyond Grade 10 (Standard 8). This could be due to the methods of teaching used in previous grades, thus contributing to the high failure rate (NCTM, 2000). For example, research has found that teacher-centred instructional methods used in the teaching of mathematics in Zimbabwe, do not generally encourage a conceptual understanding of mathematics. This ultimately results in a fairly high failure rate in mathematics in the country (Nyaumwe et al., 2004). Studies have also shown that teachers are challenged in the teaching of mathematics in Jordan, with factors including dwindling students' motivation, vague teaching strategies and ineffective problem-solving strategies (Altakhayneh, 2020).

In South Africa, where mathematics teaching is poor, the Motheo District of Education in the Free State Province is no exception (Tachie & Molepo, 2019). It is believed that metacognition in mathematics lies at the heart of learning since it (metacognition) seems to be one of the most important predictors of mathematical performance globally to develop learners' cognitive skills such as increasing the capacity of thinking, decision making and problem solving to enabling them to cope with the problems they may encounter in their daily activities (Akaydin, Yorulmaz, & Cokcaliskan, 2020:159). This research focused on the teaching of mathematics in the senior and Further Education and Training (FET) phase (Grades 7-12) of rural schools in South Africa and sought to investigate how teachers' pedagogical content knowledge practice and beliefs about mathematics' teaching and learning (Tambara, 2015) could be improved using metacognitive (MC) skills and strategies in the application of student learning. Tambara, (2015) contends that this would enable the learners to make effective use of their reasoning skills, concepts, facts, and mathematical tools in solving everyday word problems in mathematics.

The acquisition of mathematical knowledge is vital for daily transactions globally in the 21st century as well as for the 4th industrial revolution (Tachie, 2019). Therefore, the teaching of mathematics should involve relevant skills and strategies that reflect an understanding of important concepts for ordinary people so as to understand and enjoy mathematics (Penprase, 2018; Schwab, 2016). The teaching of mathematics is a fundamental process that should demonstrate both subject content knowledge (SCK) and the relevant pedagogical content knowledge (PCK) (Shulman, 1986 in Ngcoza, 2013). Being equipped with such knowledge enables teachers to be more effective, flexible, fluent thinkers, and confident in their use and application of knowledge and processes (Rigelman, 2007). Mishra and Koehler (2006) concur with this when they argue that teaching is a complicated practice requiring an interweaving of many aspects of specialized knowledge. That is why certain researchers believe that mathematics is ubiquitous to professionals in different disciplines (Anthony & Walshaw, 2009; Ngcoza, 2013). Consequently, the subject and pedagogical content knowledge (Koehler & Mishra, 2009; Ngcoza, 2013; Shulman, 1986) of mathematics should, therefore, be improved through the acquisition and development of relevant skills and strategies. In a highly technological, economy-driven world, mathematics education, as well as learners' performance in the subject, are key to admission into higher learning, placement, and employment. This has brought the study of Visser et al. (2015) to the fore: they argued that in the mission to improve mathematics performance among learners in the classrooms, both policymakers and teachers need to identify the factors that are currently influencing learners' performance in mathematics. In so doing, teachers should further establish how strong these factors are in impeding learners' progress in mathematics and should also identify the relevant skills and strategies to overcome them. As noted by some researchers (Altakhayneh, 2020; Ojose, 2011; Webb, 2010), mathematics plays a vital role in the lives of people; therefore, the approach to the teaching of mathematics requires skills, insight, dedication, hard work, and time in and out of the classroom (Myerson et al., 2007) for both teachers and learners.

Related studies have shown that problem-solving strategies like the use of acronyms, the introduction of open-ended mathematics questions and engaging learners in mathematics discussions help to develop the metacognitive abilities of learners in the learning of mathematics (Tachie, 2019; Uberti et al., 2004). These strategies help to provide learners with a list of steps to follow when working through problems whereby learners generate lists "from scratch" or alter the steps of existing lists; this action forces them to consider the information from different perspectives to create different opportunities to engage in mathematics talk either in pairs or groups for the development of their metacognitive skills and strategies, instead of teacher-directed whole-group structures. Learners should always be encouraged to assess their ability to communicate with each other and help them develop an awareness of high-quality mathematics communication skills connections that assist in developing their reasoning skills, and which will equip them to tackle any type of mathematics problems with ease.

The mathematics teacher should be concerned with each learner as an individual and should cater to his/her particular ability and aptitude, especially in practical activities when it is possible to arrange work on an individual basis. The teachers' way or method of teaching mathematics should be a practical way of teaching mathematics, and he/she should be looking at his/her own work as a way of checking and making sure that it is really what he/she would want it to be. Teachers should incorporate skills and strategies that support learners' understanding and performance. However, research on teachers' pedagogy has mostly concentrated on pedagogical content knowledge (PCK) in general but has failed to address issues on how to develop pedagogical content knowledge of teaching mathematics using meta-cognitive skills and strategies application (Tachie, 2019). This has therefore created room for investigation into how teachers' pedagogical knowledge can be improved with metacognitive skills and strategies applied to the teaching and learning of mathematics, in relation to the revised curricula, and the implications of these integral skills and strategies for learners' performance in mathematics.

Background

The importance of mathematics and how learners perform at both local and international levels has consequently been of great concern to many departments of education in various countries (Ercikan et al., 2005; Sweeting, 2011). Teachers' professional competence in teaching the subject ought to be exemplary and reflected in performance. This means that teachers' methods or ways of teaching mathematics should not be underestimated but should be of vital importance to develop learners' mathematical ability. Pre-service training in colleges and universities is important in developing a teachers' learning area both from a content knowledge perspective as well as a pedagogical one. Kilpatrick et al. (2001, p.373) argue that "teachers with a high sense of efficacy appear to be more confident in the classroom, more positive and less critical with their students; better classroom managers, more accepting and effective in responding to challenges from students (for example, why are we learning this?), and to be more effective in supporting growth and achievement." A teachers' sense of efficacy, which has to do with pedagogy, and the importance of preparing teachers by guiding them in acquiring and developing relevant knowledge, skills, and strategies are pertinent to teaching with confidence and effectiveness, thus developing subject and pedagogical content knowledge for the teaching of mathematics.

Studies conducted in South Africa have shown that the way teachers are trained during pre-service training and during in-service workshops, and the methods used by teachers in mathematics classrooms create challenges which call for a change in the teaching and learning process (Tachie, 2019). It has been widely recognised that learners are exposed to traditional methods of teaching which emphasise and encourage memorisation and application of algorithm and procedures (calculations through the accepted standard of rules or formulae) and do not incorporate the use of metacognition and metacognitive strategies in the mathematics classrooms. The training of teachers either during pre-service or during in-service workshops should incorporate new skills and strategies (metacognitive skills and strategies) of teaching mathematics; this should be equivalent to the international levels in order to improve learner performance.

The Curriculum and Assessment Policy Statement (CAPS) introduced in 2011 in South African schools, has prescribed a shift from traditional modes of teaching to a learner-centred approach, which focuses on the nurturing of proficiency and development of mathematical practices (DBE, 2011; Sanni, 2007). The policy explains that the teaching and learning of mathematics should develop "a critical awareness of how mathematical relationships are used in social, environmental, cultural and economic relations, confidence and competence to deal with any mathematical situation without being hindered by a fear of Mathematics, an appreciation for the beauty and elegance of Mathematics, a spirit of curiosity and a love for Mathematics, recognition that Mathematics is a creative part of human activity, deep conceptual understandings in order to make sense of Mathematics and the acquisition of specific knowledge and skills necessary for the application of Mathematics to physical, social and mathematical problems" (DBE, 2011, p.8).

To effect a change in teachers' practices, Sanni (2007) advocates for the necessary support and assistance for such desired changes to become established in the classroom. The support demanded by Sanni (2007) included the presentation of lessons that support the use of metacognitive skills and strategies so as to nurture proficiency and development of mathematical practices in learners' problem-solving.

Mathematical knowledge includes mathematical ideas; the type of mathematics that unlocks how mathematical knowledge is produced or discovered (Kilpatrick et al., 2001). However, as previously stated, acquiring mathematical knowledge also involves more than having knowledge of mathematics; Schuck (1999) confirms the necessity for teachers to not only have a solid subject content knowledge where concepts are correctly understood and procedures performed without fault, but also the need for teachers to investigate the conceptual foundations of that knowledge. In addition, teachers must also understand mathematics in ways that allow them to clarify and unpack ideas in a particular manner (Kilpatrick et al., 2001) which gives them the vital pedagogical content knowledge needing to be applied in the classroom.

Divergent teaching methods foster teachers' expertise in teaching mathematics to enable them to train their learners to function adequately in the future workplace (Kember et al., 2006). Methods of teaching mathematics should incorporate strategies such as instructional designs, which embed learning in meaningful contexts such as problem-based learning whereby learners are offered the opportunity to devise strategies to help them find solutions to problems. This ought to be aligned with the growing need for the developing of competencies, such as critical thinking in problem-solving through the use of metacognitive skills and the ability to solve novel problems (Kember *et al.*, 2006). However, cognitive outcomes demanded in the curricula are not always displayed in teaching (Darling-Hammond et al., 2020).

In many instances, teachers have been criticised for not applying skills and strategies that support the development of learners' problem-solving expertise in the mathematics classroom (Boyatzis et al., 2002). Many years ago, Shulman (1986) identified a model of pedagogical reasoning which comprises a cycle of several activities such as comprehension, transformation, instruction, evaluation, reflection, and new comprehension. The author contends that "if beginning teachers are to be successful, they must wrestle simultaneously with issues of pedagogical content (or knowledge) as well as general pedagogy (or generic teaching principles)" (Shulman, 1986, p.4), which emphasises an aspect important in the current era of the 4th Industrial Revolution. Developing learners' mathematical proficiency is, therefore, what each professional mathematics teacher aims to do (or should aim to do).

Anthony and Walshaw (2009) affirmed that in the teaching and learning of mathematics, issues of research-based assessment and monitoring, promoting learner engagement, and creating a conducive classroom environment for learning should be a priority for every mathematics teacher. This can only help teachers to achieve better outcomes in teaching and learning in schools. The Research Hypothesis (Ho) is: Teachers who understand the concept of metacognition do better with metacognitive practices in their teaching in their classrooms.

Conceptual Framework of Metacognition and Pedagogy

According to Flavell (1976), metacognition is the process of monitoring, planning, controlling, and evaluating information to achieve a goal. In this research, the concept of metacognition is viewed as a cognitive series of processing information that allows a teacher to use his/her previous knowledge acquisition and experiences in an organised manner to assist learners in problem-solving by selecting, seeking and applying relevant skills like reflection and evaluation. Metacognition, a process by which an individual is aware of his/her own brain processes that occur during learning, is learning to think about the how and why of what one does.

Research on pedagogical knowledge and metacognition worldwide has established that many curriculum documents reflect on the importance of metacognition application for effectively improving learners' performance in the learning situation, but fail to address how different, or how specific metacognitive skills and strategy application help in improving teachers' pedagogical knowledge in the teaching of mathematics in rural areas (Beswick, 2014, Wilson 1998), hence the current research. The use of metacognitive skills and strategies, as proposed by some researchers in mathematics teaching and learning, helps to facilitate learners' understanding of the lesson and further increases their awareness of what they know (and do not know) based on a certain area of instruction (Beswick, 2014; Cao & Nietfeld, 2007).

According to Harris et al. as cited in Waters and Schneider (2010, p.227), the use of metacognitive skills and strategies helps to improve teachers' methods of teaching and their learners' understanding of a lesson which thus influences the lives of their learners. In applying metacognitive skills and strategies to class activities, teachers' pedagogical knowledge of teaching and learning of the subject, knowledge of classroom assessment to be done during teaching and learning, knowledge of psychological components, knowledge of individual differences, as well as the knowledge of classroom management have an influence on the mathematics teaching and learning, thus resulting in a better understanding by learners. In addition, using learners' evaluations and their views on teaching and learning helps in informing possible changes to a module framework or content.

Mathematics is seen as a combination of calculation skills and reasoning (Hannula et al., 2004). Shepherd's (2012) investigation into the effect that teachers' subject knowledge has on learner performance in South African schools revealed that deep knowledge and understanding of the subject matter taught are important, but of greater importance is the ability of the teacher to transfer that information in a meaningful way to learners through the use of appropriate skills and strategies. This, therefore, has created room for further investigation to establish the types of knowledge, skills, and strategies mathematics teachers employ as their pedagogical knowledge in the teaching of mathematics (Shepherd, 2012).

For this reason, mathematics' teaching and learning should always incorporate skills and strategies that facilitate learners' thinking abilities and reasoning. Better performance in mathematics depends on an individual's mathematical understanding, concentration and confidence; however, this depends on a teacher's know-how to direct and connect his/her learners' thinking abilities and critical reasoning through logical ways of presenting the lesson (Jagals & Van der Walt, 2013) and incorporating different perspectives, such as higher-order thinking skills.

Teaching and learning is a complex process whereby a general teaching method is brought into question, reflected upon, and proven inconclusive (Jagals, 2015). This study employed a bricolage conceptual framework, comprising different theories in order to achieve the purpose of the study. Through a bricolage conceptual framework, certain principles of the constructivist-learning/teaching approaches were used that helped the researcher in conceptualising this research. Mahlomaholo (2013) described a bricolage framework as multiple voices that aim to establish solutions without restricting a researcher to one theory but its collaboration aims to achieve one objective. A bricolage model promotes hands-on activities to establish something of importance to human dignity (Chevalier & Buckles, 2019) that is not only based on theory but practical work that correlates mathematical concepts to real-life situations. For instance, making use of physical objects in teaching and learning methods that always incorporate skills and strategies to facilitate learners' thinking abilities and reasoning in mathematics classroom becomes exciting which creates a conducive learning environment and enables multiple methods of teaching and learning of mathematics. This model is able to provide a detailed account of concrete thinking which is an element of metacognition that enables the researcher to utilise a more hands-on activity-driven rather than a procedural way, of teaching and learning. This is exactly what the study is keen to achieve for the benefit of both the teachers and ultimately, learners in the mathematics classroom. The bricolage model becomes a bridge of incorporating various activities which is grounded on creativity, collaboration, knowledge, respect and a vision (Wise, 2017). One of the central claims of this framework is based on heterogeneous tools and materials, which implies that teachers are free to develop their pedagogical knowledge of teaching mathematics through the use of a range of teaching methods and varied teaching aids that meet the diverse needs of learners.

Theoretical Framework

The research used metacognitive theories proposed by Schraw and Moshman (1995), to understand how individuals combine different kinds of metacognitive knowledge and regulatory skills into systematised cognitive frameworks in order to develop their pedagogical knowledge. Schraw and

Moshman (1995) emphasise aspects of components of metacognitive knowledge such as conditional knowledge (what one knows about cognition) and metacognitive regulation or control processes such as comprehension monitoring (how one uses that knowledge to regulate cognition). The systemic integration of these components, such as theoretical knowledge about variables, affects cognitive performance (Schraw & Moshman, 1995). Schraw and Moshman (1995) contend that the use of metacognitive theories helps a teacher or researcher distinguish between metacognitive knowledge and regulation. These aspects/concepts are worth taking seriously by teachers during teaching to help improve their method of teaching since traditional mathematics teaching emphasises memorisation and application of algorithm and procedures which do not support the move to a learner-centred approach. In this way, learners are not given the opportunity to engage in and demonstrate their views, thus, making the move away from procedural fluency to conceptual understanding of how certain things work the way they do (Jacobs & Paris, 1987; Kilpatrick et al. 2001; Tachie, 2019).

Declarative knowledge is about learning materials or about the behaviour of the teacher or the method of teaching. Teachers should be mindful of the kind of learners they are dealing with as well as the knowledge to be applied at a certain stage in order to execute a task, since completion of the task correlates with the kind of knowledge exhibited by the learner. Abraham (1997) refers to this knowledge in his second stage, as the exploration stage, in which learners are placed in groups, given opportunities to explore the question(s) or problem(s) given to them in order to gather the relevant skills/procedure that help to generate answers for the question(s)/problem(s). At this level, the teacher's role is to act as facilitator, providing scaffolding for the learning, guiding the process, posing pertinent questions to channel learners' explorations and providing cues about how to proceed, without showing learners "exactly how to go about solving the problem" (Abraham, 1997; Stein et al., 2001). This is a vital period for the teacher since the selection of correct procedure links to the correct application of metacognitive skills and strategies and also leads to the development of pedagogical knowledge (Tachie, 2019). The above knowledge also connects to the conditional knowledge, which refers to knowing when and why to apply various cognitive actions (Garner, 1990; Lorch et al., 1993). The pedagogical knowledge of a teacher may influence conditional knowledge since a teacher's directives for learners regarding "why and when" to apply various cognitive actions result in fruitful output in the teaching and learning situation. This, according to Kramarski and Kohen (2016) and Tachie (2019), improves the metacognitive abilities of the teacher, which, according to other researchers, support teachers' pedagogical knowledge of teaching mathematics (Shulman, 1986; Ni Shuilleabhain & Seery, 2018). The teacher's way or method of teaching mathematics should be a practical way of teaching mathematics, and he/she should be looking at his/her own work as a way of checking and making sure that it is really what he/she would want it to be.

The main scholarly question in this article is: *In what ways do teachers apply metacognitive skills and strategies to develop their pedagogical knowledge of teaching mathematics?* To address the above question effectively, metacognition and its associated strategies and skills are discussed, offering an understanding of what all it entails and how the application of those skills and strategies assist in developing teachers' pedagogical knowledge of teaching mathematics**.**

Research Methodology

The design for this research approach is an explanatory sequential mixed-method design which is a survey as well as interviews. The research took the form of a descriptive survey as well as a case study. The first phase dealt with the collection and analysis of the quantitative data on teachers' opinions of how they taught mathematics for the benefit of learners in their schools. This was followed by the collection of qualitative data based on emerging issues from the quantitative outcome (Creswell et al., as cited in Tashakkori & Teddlie, 2003). Qualitative data was sought to complement the quantitative data and focused on teachers' teaching strategies/skills during observation of their teaching of mathematics in their classrooms. In this design, one set of data thus complemented the other, helping to overcome any weakness associated with each (Creswell & Plano-Clark, 2007). The use of this method was informed by the notion that surveys are usually used where opinions are required (David & Sutton, 2004); this allows researchers the opportunity to gather data at a particular time in order to describe the nature of existing conditions, or to identify standards which can be used to compare or determine the relationships between specific events (Cohen et al., 2007).

The case study, on the other hand, helps the researcher to glean a deeper understanding of, or reasons for, the phenomenon being investigated. It further reveals the complex textual descriptions of how people experience a given issue (Maree, 2010): in this case, how teachers' application of metacognitive skills and strategies help reveal the improvement of their pedagogical knowledge of teaching mathematics in rural schools of the Free State Province, hence the use of both methods. For the quantitative data the Cronbach alpha measure of internal consistency was calculated to measure the reliability of the questionnaire. The overall Cronbach alpha value was 0.60, showing that the questionnaire was valid and reliable (Goforth, 2015). For the qualitative data, trustworthiness was ensured by sending the transcriptions of the interviews to the participants for member checking in which the researcher's aim was to ascertain the actions of the mathematics teachers' application of metacognitive skills and strategies in their teaching, and how it supported or improved, their pedagogical knowledge of teaching mathematics. It further ascertained how teachers' pedagogical practices and beliefs about learners' learning, specifically related to a revised curriculum, were impacted as a result of their participation in applying metacognitive skills and strategies used in their teaching and how they applied the above for the benefit of their learners in the teaching and learning of mathematics.

Sample

The sample used for this study comprised in-service teachers. For the quantitative part, forty-eighty participants (N=48 teachers: males=21 and females=27) were sampled, based on the cluster sampling technique, to take part in this study by completing the questionnaire. The rationale for this was to check teachers' opinions of how they taught mathematics for the benefit of learners in their schools. For the qualitative part, six teachers (N=6 teachers; males=3 and females=3) were purposefully selected, and their teaching strategies/skills were observed during their teaching of mathematics in their classrooms. The selected participants comprised senior and FET phase (Grades 7-12) mathematics educators. Their ages ranged from 26 to 51 years and above, while their experience in the teaching fraternity ranged from 3 to 21years above. This sample included both male and female educators from about twenty (20) selected schools, who were teaching Grades 7 – 12, and were therefore representative of the population for the study. All ethical considerations were observed prior to commencement of the fieldwork. The reliability and validity of the instrument used are also outlined. A pilot study was also conducted.

Instruments

Questionnaires, which consisted of closed-ended items, class observation, and interviews were the instruments used for this research. The rationale was to check how the method(s) used, with the help of textbook(s), provided guidance to inform which mathematics aspects were taught and how this was presented to the learners. The items focused on teachers' methods and strategies they used in their teaching and how they went about their teaching of mathematics for the development of their pedagogical knowledge of teaching mathematics.

The second instrument was a classroom observation schedule used during observation of teachers in their mathematics class. The final instrument was the interview schedule used when conducting individual interviews with teachers after observation of their lessons.

Procedure

Firstly, prepared questionnaires were given to the respondents in order to elicit their opinions on the method(s) used in the teaching of mathematics. One hundred close-ended questionnaires were personally distributed by the researcher and his research assistant but only forty-eight were completed. A sample should be representative and thus identify findings that are probably generalizable to that target population. The number of respondents may affect the generalization of the findings. The results are critical but they may not be generalised to the large scale due to low response rate as the aspect of representativeness could be affected. If representativeness cannot be maintained, it will affect the generalization of the findings. I do acknowledge the fact that even through the respondents/responses were so low, but the findings cannot be dismissed based on low respondents. The researcher also conducted interviews based on teachers' lesson presentations and views on metacognition after an analysis of the responses to the questionnaires. All ethical formalities were observed in carrying out this research.

Data Analysis (Quantitative)

A statistical test was applied to obtain the p-value. Thus, in an attempt to ascertain pedagogical practices and beliefs of teachers in their teaching practice and when applying metacognitive skills and strategies, the researcher applied a T-test. Statistical software, SPSS Version 20 (R software), was used. The T-test was applied by first formulating the null-hypothesis and the alternative hypothesis. Research Hypothesis (Ho) was: Teachers who do not understand the concept of metacognition do not apply metacognitive practices well in their classroom teaching. Alternative hypothesis (H1): Teachers who understand the concept of metacognition do perform better with metacognitive practices in their classroom teaching. The hypothesis can be accepted or rejected based on a particular α level of 0.05 or level of significance. When the calculated p-value is less than the level of significance (α), the null hypothesis (Ho) is rejected in favour of the alternative hypothesis and it is concluded that teachers who understand the concept of metacognition do better with metacognitive practices in their classroom teaching. However, when the calculated p-value is greater than the level of significance, then the alternative hypothesis is accepted and concluded that teachers who do not understand the concept of metacognition do not perform better/well with metacognitive practices in their classrooms teaching. Metacognitive practices here is an association between the variables, hence when the calculated p-value is greater than the level of significance (α), the null hypothesis (Ho) is accepted and it is concluded that there is no association between the variables. It was ensured that the probability of making type 1 error was very small since the calculated p-value of 0.04 was less than the level of significance of 0.05.

Table 1:

Skills and strategies identifying/showing teachers' application of metacognition

Question	Mean	Std.Dev	P-value	Ranking
I select relevant teaching and learning materials/resources relevant to the topic to be taught.	3.667	0.595	0.000	1
I consider learners' prior knowledge first before teaching any new topic/concept.	3.604	0.736	0.000	2
I consider my learners' cultural background.	3.591	0.693	0.000	3
I always plan my lesson well before I go to class to ensure that I provide relevant information for my learners in class.	3.521	0.743	0.000	4
I pay special attention to learners with barriers to learning.	3.500	0.825	0.000	5
I solve all the mathematical problems myself as I prepare the answers in at least two different ways.	3.375	0.815	0.000	6
I monitor learners' discussions and give input where necessary during problem solving.	3.354	0.812	0.000	7
I encourage my learners to use different strategies in solving mathematics problems.	3.333	0.753	0.000	8

Question	Mean	Std.Dev	P-value	Ranking
I welcome learners' contribution during lessons.	3.313	0.776	0.000	9
In general, how frequently do you apply metacognition in your teaching?	3.271	0.792	0.000	10
I consider my learners' mathematics anxiety.	3.271	0.893	0.000	11
I probe my learners to explain their thinking/answers.	3.256	0.848	0.000	12
I consider my learners' emotional responses.	3.250	0.838	0.000	13
While I am teaching, I allow learners to reflect on their thinking.	3.188	0.790	0.000	14
I guide my learners to engage themselves in problem solving activities.	3.146	0.714	0.000	15
When learners do not understand certain concept(s) during teaching and learning, I change my teaching strategy.	3.087	0.839	0.000	16
I re-teach the lesson if learners did not understand it.	3.083	0.739	0.000	17
I help learners to discover their own level of understanding in mathematics.	3.083	0.739	0.000	18
I pause to think about what I have taught during the lesson.	2.935	0.680	0.000	19
Kindly explain your response to the level of perfection at how you apply metacognitive skills and strategies in your teaching and learning of mathematics at your school.	1.234	0.560	0.000	20

Table 1 indicates teachers' awareness of the concept of metacognition. The largest group 39(81%) of the respondents indicated that they were aware of the concept. Only 9(19%) indicated that they were not aware of the concept. Further from Table 1, 27: (52%) of the respondents indicated that they have heard of the concept metacognition from a lecturer who had conducted a workshop. This is followed by 7(15%) who indicated that they heard the concept from both a lecturer who conducted a workshop as well as from the internet whereas only 3:(6%) of participants indicated they heard of the concept of metacognition from the internet.

Table 1 also indicates the skills and strategies of identifying/showing teachers' application of metacognition. For each statement relating to skills and strategies identifying/showing teachers' application of metacognition in the teaching of mathematics, their scores or views are outlined below.

In comparing the means of the items in the questionnaire, it was observed that different items had different means and items were reduced into rankings. A p-value of 0.04 (compared to a significance level of 0.05) rejected the null hypothesis (H_0); therefore, there was a statistical association between teachers who understand the concept metacognition and employed better metacognitive practices in their teaching. In other words, responses in the questionnaire depended on the way teachers understood and applied the concept of metacognition and how it supported their teaching practices in the teaching of mathematics in schools.

The alternative hypothesis (Teachers who understand the concept of metacognition apply better metacognitive practices in their teaching), as well as the research question, (*In what ways do teachers apply metacognitive skills and strategies to support the development of their pedagogical knowledge of teaching mathematics?),* were intended to establish the ways in which teachers applied metacognitive skills and strategies to support the development of their pedagogical knowledge of teaching mathematics. The ability to integrate these two were critical for successfully answering the research question for the study. Table 1 gives a summary of the skills and strategies mostly employed by teachers in their teaching and learning indicating their pedagogical knowledge of teaching mathematics. Table 1 further depicts that there were differences in responses given by the participating teachers in applying metacognitive skills and strategies in mathematics problem-solving.

Table 1 also depicts the views of the majority of the respondent teachers on how their understanding of metacognition helped them to apply those skills and strategies to improve their mathematics' teaching and thus learners' learning of mathematics. The views of the larger group of respondent teachers illustrate that incorporating the use of metacognitive skills and strategies to in their teaching of mathematics improves their pedagogical knowledge. The application of metacognitive strategies is illustrated below, for example. When learners do not understand a certain concept(s) during teaching and learning, respondents indicated that they use various skills and strategies: "I change my teaching strategy" (75%), "I pause to think about what I have taught during the lesson" (75%), "I probe my learners to explain their thinking/answers" (71%), "I consider my learners' emotional responses" (79%), "I pay special attention to learners with barriers to learning" (79%). Others include "I consider my learners' cultural background" (83%), "While I am teaching, I allow learners to reflect on their thinking" (82%), "I re-teach the lesson if learners do not understand it" (81%), "I solve all the mathematical problems myself as I prepare the answers in at least two different ways" (79%) and "I guide my learners to engage themselves in problem-solving activities" (81%).

Qualitative Data Analysis

In the presentation of the data analysis codes such as TA, TB, TC etc., were used to represent Teacher A, Teacher B, Teacher C respectively and so on. Content analysis was used on the observations data and the interview transcripts for the qualitative part. After verbatim transcriptions of the interviews were completed, the data were further examined in-depth with repeated readings of the transcriptions. Data were analysed manually with the assistance of experts within the learning area through content analysis. The texts were segmented into codes, which were classified into categories and grouped into themes for interpretation and reporting (MacMillan & Schumacher, 2010). Thematic analysis was then used to analyse the collected data.

Discussion of Findings

This section was specifically formulated to describe and portray the characteristics of the respondents in areas such as race, gender, phase teaching, the highest level of qualification, the total number of years of teaching mathematics, teachers' awareness of the concept metacognition and explanation of response to metacognition. Table 1 summarises the participants' biographical information.

Since the respondents were selected randomly for the study, 21 males and 27 females were selected; thus 44% were males, and 56% were females in the sample, with the majority of the participants being female. The table also shows that the major racial groups participating in the study was Africans with a total number of 16(33%), followed by the White 11(23%) and Coloured 11(23%) group of teachers respectively. The Indian group was the smallest group of respondents, totalling 10(21%). Table 1 further displayed the different phases in which the respondent teachers teach mathematics. The table depicts the Foundation Phase as the phase in which the majority of the teachers teach with 13(27%). This was followed by Intermediate and FET Phase with 12(25%) teachers, respectively. The smallest group of teachers was found in the Senior Phase, with 11(23%) teachers. The table also shows that the highest qualification amongst the respondent teachers was M.Ed in mathematics held by 4(8%) teachers. Most teachers held either BEd (Hons) (18 - 38%) or BEd in mathematics (19 – 40%) with much smaller numbers being more qualified (4 – 8% MEd) or less qualified (4 – 4yr teaching diploma, 3 – 3yr teaching diploma). Based on the mathematics teaching experience, the largest group 16(33%) of respondents indicated that their total years of teaching mathematics is below 5 years, whereas 12(25%) indicated that they have been teaching 16 years and more. Some 11(25%) respondents indicated that they have been teaching for more than 11years, and 9(19%) respondents had taught mathematics for more than 10 years.

In this study, t-tests was used to checked whether responses to individual questions deviated from pure chance, and the p-values were small. The conclusion was that the average response differed from the expected central value and thus it is concluded that the questions were answered deliberately. Additionally, the mean differences were all in a positive direction.

Then, separately, the hypothesis "teachers who understand the concept of metacognition apply better metacognitive practices in their teaching" was evaluated via a series of unbalanced ANOVAs. For each scaled question in the questionnaire, the question of whether the respondent was familiar with the concept of metacognition was used to split the responses. The average response among those who stated they were familiar with the concept was compared to the average response among those who stated they were not familiar with the concept. The p-value which was found to be 0.04 provides an evidence that the null hypothesis does hold and hence metacognitive practices are applied by those familiar with the concept. The family-wise error rate was protected using the Holm-Bonferroni method (Holm, 1979).

A congeneric reliability check was done and resulted in a high value of 0.98. The respondents showed knowledge of metacognition and indicated its application in their teaching practices in the classroom. It emerged from the analysis that most respondents who were aware of the concept of metacognition, demonstrated better reasoning skills and orderly presentation of their lessons/works in comparison to their counterparts who had never heard of the concept. This supports Schraw and Moshman (1995), who theorise that individuals consolidate different kinds of metacognitive knowledge and regulatory skills into systematised cognitive frameworks in order to develop their pedagogical knowledge. In the same vein, Jansen (2011) is of the view that if a teacher does not know enough mathematics, he/she is not able to teach the subject effectively, irrespective of the generic training inputs offered by the relevant authority through the workshop and in-service training programmes. Further explanation from the respondents on where they heard the "concept of metacognition" also showed an edge over their counterparts in their application of metacognitive skills and strategies in their lessons to solve the problem.

Teachers' pedagogical knowledge is developed through the incorporation of metacognitive skills and strategies which assist learners far better than those who do not have such skills.

The pedagogical knowledge of a teacher is said to be a significant part of the teachers' knowledge base in order for them to be effective in choosing worthwhile tasks for their learners (Poulson, 2001). The way teachers solve mathematics problems with learners is vital in the learning of mathematics as it helps them in their meta-cognitive skills acquisition in mathematics problem-solving. This is in line with the findings of a study conducted by Altakhayneh (2020) in Jordan, which revealed challenges faced by teachers in teaching mathematics. Altakhayneh (2020) highlighted decreasing levels of motivation among students and inadequate teaching strategies used in the teaching of mathematics and ineffective problem-solving strategies as these challenges. Engagement of learners in teaching is a key component of learning which some teachers have been implementing. However, the majority of teachers lack certain requisite skills and strategies to engage learners in effective teaching and learning of mathematics. Without engagement, there is little motivation for learners to learn. In some cases, the teacher might be a mathematics genius in the teaching and learning fraternity and a specialist in that field, but if he/she possesses no sound teaching methods, that is pedagogical content knowledge, that help shape classroom mathematics teaching and learning by raising the interest of the learners in the subject, he/she will not be able to reach the learners, and thus no effective learning can take place in the classroom, thereby discouraging learners (Cooney, 1999; Machaba, 2016).

Through the use of metacognitive skills and strategies, teachers are able to initiate some ideas that help build on the prior knowledge of their learners when teaching some of the mathematics topics in class during the period of this study. According to the teachers, this actually supports the understanding of the learners and further boosts their participation in the lesson in mathematics problem-solving. In addition, this encourages the teachers to confirm that their pedagogical knowledge has improved. Clearly, teachers' pedagogical knowledge of teaching mathematics improves when they incorporate different skills and strategies (metacognitive skills and strategies) in their teaching and learning; the activities given by the teachers are a clear indication of what the teachers are doing to improve the application of their pedagogical knowledge.

In the interview question: *How often do you involve your learners in your teaching and learning, and what was the effect*? participant teachers indicated that in most cases, they involved learners in teaching and learning in class; for example, when teaching a topic such as numeric and geometric number patterns, as indicated by some of the teachers, one teacher used materials such as bottle tops. According to the teacher, learners were happy when she used such materials, and when she involved them in handling manipulatives: *Learners appeared very enthusiastic and contributed well to the teaching and learning of the topic the moment I used bottle tops and called them to come and show me how to use it* (Teacher B). Teacher E indicated that *collaboration with our learners is very important in teaching and learning of mathematics, especially how interaction with learners assist in improving our teaching and learning in general.*

The observations revealed how the teachers moved around in the class, checking and monitoring their learners' participation and progress in their group discourse. How learners put questions to each other was also well monitored. Teachers need to apply all relevant skills and strategies to stimulate learners' curiosity and interest in problem-solving through learning.

The study revealed that when teachers utilize their pedagogical knowledge and apply metacognitive skills and strategies in their teaching practice, it helps their learners learn more effectively especially when they are actively engaged. This means that when teachers' pedagogical knowledge is improved by a greater understanding of metacognition, it facilitates learners' learning. Thus, learners' understanding of mathematical concepts are enhanced, according to Kazima et al. (2008), when learners have opportunities to undertake tasks similar to those undertaken by professionals within a discipline. Kazima et al. (2008) and Sheinuk (2010) put forward the concept of a teacher's ability to "hold and use mathematics". Apart from being able to do the mathematics in the teaching and learning situation, a teacher needs to have absolute clarity, through relevant skills and strategies, about the goals of a mathematics lesson using approaches that transmit ideas and concepts into reality. These approaches, identified by the teachers on a scale of 1 to 4, include teachers' thinking about their teaching. This approach opens up an opportunity for the teacher to create an environment conducive to engaging learners in a variety of relevant activities that generate curiosity and interest where much of the attention is placed on investigation (Abraham, 1997). Teachers' level of achievement in mathematics usually affects how they respond to mathematics and what they believe about the subject, which has consequences for his/her learners in the classroom situation (Brady & Bowd, 2005).

Teachers' success or failure in formal mathematics education experience contributes to how confident they are in teaching the subject, and this influences the performance of the learners through the application of metacognition (Brady & Bowd, 2005; Cao & Nietfeld, 2007; Beswick, 2014). Mwamwenda (1995) argues that adequate teacher preparation in terms of lesson preparation and presentation makes the teachers more effective in the classroom. In other words, the more prepared teachers are, the more confidence they display in presenting their lessons to the learners. When teachers are adequately prepared for their lessons, they are able to involve their learners in teaching and learning, pay attention to their contributions, monitor their discussions and give input where necessary, and are able to select relevant teaching and learning materials/resources relevant to the topic being taught, in order to win the confidence of the majority of the learners in the classroom through active participation and discussion (Hamacheck, 1990; Reynders, 2014; Tachie, 2020). This is confirmed through the interviews when the teachers indicated that in using different strategies during teaching and learning it developed an understanding of concepts for the learners. This includes involvement of learners in teaching and learning in class whereby different materials such as manipulatives in the teaching of concepts like numeric and geometric number patterns are used. This, according to some researchers such as Ojose (2011), Stacey and Turner (2015), and Suciati et al. (2020), help teachers to overcome their problem of teaching mathematics. This preparation, the subject content and pedagogical content knowledge, helps teachers assist learners in formulating, using, and interpreting mathematics in various contexts.

Conclusion

The results from the research suggest that most teachers in this study incorporated many meta-cognitive skills and strategies (such as planning, monitoring, and reflection) which improved their pedagogical knowledge, and this further assisted the mathematical problem-solving skills of their learners. This was done through the help of teachers' questioning methods, monitoring of learners' discussions, supporting learners' trial-and-error methods, and spontaneously thinking about their own thinking (Flavell, 1976; Sepeng, 2010). As a result, Tachie's (2021) proposed metacognitive framework (adopted from Hill et al. 2008, p.403) was developed (refer to Figure 1).

Contribution to the Literature

In the literature review of this study, metacognitive skills and strategies, as well as some learning skills, were outlined to enhance effective teaching and learning of mathematics in schools. The researcher suggested these skills and strategies as among factors that could be incorporated into the teaching and learning of mathematics to improve learners' performance in a global perspective. Besides the strategies outlined from literature and the recommendations of this study, there are other strategies that could be incorporated into the teaching and learning of mathematics to enhance learners' performance in mathematics. One of such is an adequate application of metacognitive abilities to improve the pedagogical content knowledge of the teacher for better teaching and learning of mathematics. If a teacher is aware of metacognition and has the ability to apply metacognitive skills and strategies his/her pedagogical content knowledge would result in better teaching and learning of mathematics. The metacognitive skills and strategies applied by teachers in the teaching of mathematics if well demonstrated and taught to the learners, can stimulate their confidence in the teaching of mathematics, which positively affects their learners. Thus, the study revealed that teachers who understood the concept metacognition apply better metacognitive practices in their teaching and vice versa. It emerged from the analysis that most respondents who were aware of the concept metacognition demonstrated better reasoning skills and orderly presentation of their lessons comparison to their counterparts who had not heard of the concept.

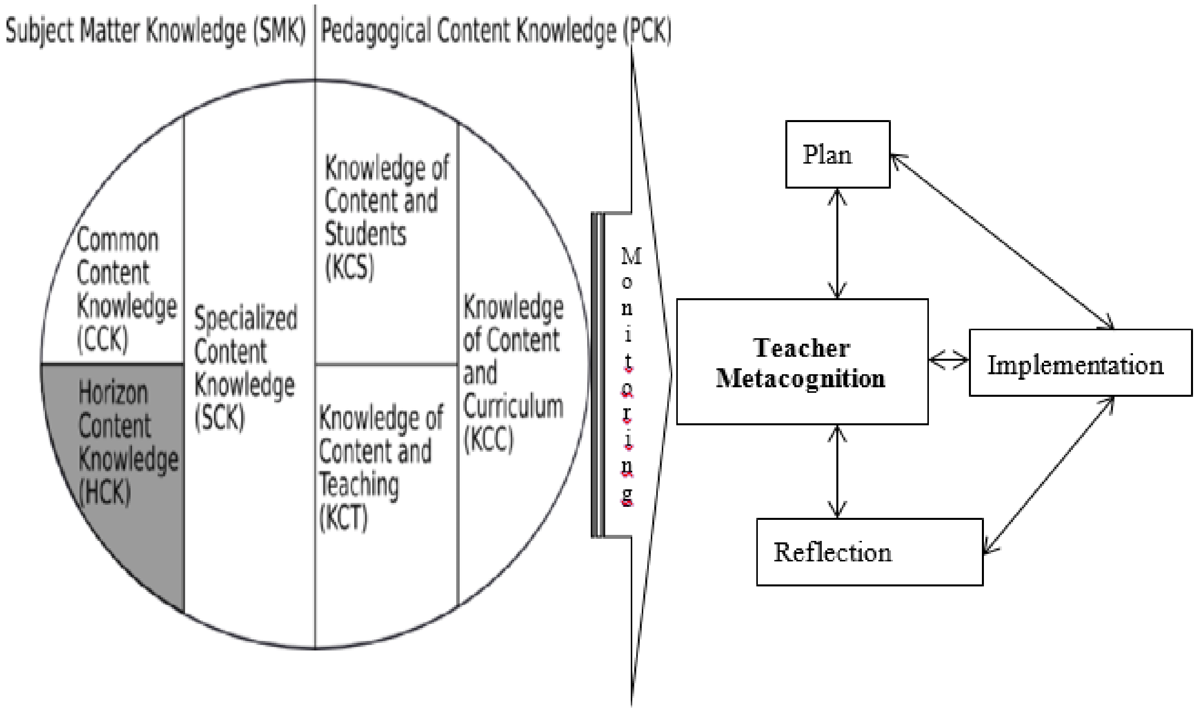

Figure 1: Tachie's 2021 proposed Metacognitive Framework (adapted from Hill et al. 2008, p.403)

Tachie's (2021) proposed framework (refer to Figure 1), premised on Hill et al.'s (2008) proposal as its bedrock, assists teachers in understanding the theory behind the teaching of mathematics in order to function effectively. Tachie's (2021) teacher metacognition framework ensures that teachers become aware of the various domains to apply metacognition. As teachers plan, implement, reflect and monitor their own teaching for effective learning, metacognition therefore plays a vital role. It is critical that teachers have this knowledge and understanding and are able to utilise it in their planning, monitoring and reflection but they have to do it within the context of metacognition. The argument here is that the application of metacognition in teachers' pedagogical knowledge of the teaching of mathematics, where teachers move from the basic knowledge into the arena of metacognition and apply it in their teaching and learning which will improve their practices. It is my belief that when teachers apply the basic knowledge of the domains derived from Hill et al. (2008), and infuse it through planning, monitoring and reflection, their pedagogical knowledge of teaching mathematics will improve while enhancing learners' mathematics understanding.

Recommendations

From the findings of this study, the following recommendations are made: It is recommended that the use of metacognitive skills and strategies for developing teachers' pedagogical knowledge of teaching mathematics can be adapted to the South African context. Universities and the Department of Basic Education could collaborate to maintain the quality of development of teachers' pedagogical knowledge of teaching mathematics through regular metacognitive skills and strategies application. This could be done effectively by engaging teachers in professional development exercises whereby teachers can observe their colleagues teaching and therefore help them to share their frustrations in a way that has the potential to lead to changes in teachers' values and conceptions of teaching and learning which impact their pedagogical practices meaningfully in the teaching and learning of mathematics in schools. Lastly, teachers should make effective use of Tachie's (2020) proposed framework in their teaching and learning to improve their pedagogical knowledge of teaching mathematics in schools globally.

Conflict of Interests

The author declares that no conflict of interest exists regarding this research work.

Acknowledgement

This section is excluded to make this manuscript anonymous.

References

Abraham, M. R. (1997). The learning cycle approach to science instruction (Research matters – To the science teacher, No. 9701). Columbia, MO: The National Association for Research in Science Teaching.

Akaydin, B.B., Yorulmaz, A. & Cokcaliskan, H. (2020). Investigation of Primary School Students' Metacognitive Awareness and Decision-Making Skill. International Journal of Progressive Education, 16(4), 157-171. doi: 10.29329/ijpe.2020.268.10

Altakhayneh, B. (2020). The Impact of Using the LEGO Education Program on Mathematics Achievement of Different Levels of Elementary Students. *European Journal of Educational Research*, 9(2), 603-610. doi:10.12973/eu-jer.9.2.603

Anthony, G., & Walshaw, M. (2009). Characteristics of effective teaching of mathematics: A view from the West. *Journal of Mathematics Education*, 2(2), 147-164.

Darling-Hammond, L., Flook, L., Cook-Harvey, C., Barron, B., & Osher, D. (2020). Implications for educational practice of the science of learning and development. *Applied Developmental Science*, 24(2), 97-140.

Hill, H. C., Ball, D. L., & Schilling, S. G. (2008). Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39(4), 372-400.

Beswick, K. (2014). What teachers' want: Identifying mathematics teachers' professional learning needs. *The Mathematics Enthusiast*, 11(1), 83-108.

Boyatzis, R. E., Stubbs, E. C., & Taylor, S. N. (2002). Learning cognitive and emotional intelligence competencies through graduate management education. *Academy of Management Learning & Education*, 1(2), 150-162.

Brady, P., & Bowd, A. (2005). Mathematics anxiety, prior experience and confidence to teach mathematics among pre-service education students. *Teachers and teaching*, 11(1), 37-46.

Cao, L., & Nietfeld, J. L. (2007). College Students' Metacognitive Awareness of Difficulties in Learning the Class Content Does Not Automatically Lead to Adjustment of Study Strategies. *Australian Journal of Educational & Developmental Psychology*, 7, 31-46.

Chevalier, J.M. & Buckles, D.J., 2019. Participatory action research: Theory and methods for engaged inquiry. Routledge.

Cohen, L., Manion, L., Morrison, K., & Morrison, R. B. (2007). *Research methods in education.* Routledge.

Cooney, A. (1999). Reflection demystified: answering some common questions. *British Journal of Nursing*, 8(22), 1530-1534.

J. W. Creswell, J. W., & Plano-Clark, V. W. (2007). *Designing and conducting mixed methods research*. Sage.

David, M., & Sutton, C. D. (2004). *Social research: The basics*. Sage.

Department of Basic Education (DBE). (2011). *Curriculum and Assessment Policy Statement (CAPS) Grades 1-3 Mathematics.* Government Printers.

Jagals, D. (2015). *Metacognitive locale: a design-based theory of students' metacognitive language and networking in mathematics* (Doctoral dissertation). North West University.

Ercikan, K., McCreith, T., & Lapointe, V. (2005). Factors associated with mathematics achievement and participation in advanced mathematics courses: An examination of gender differences from an international perspective. *School Science and Mathematics*, 105(1), 5-14.

Flavell, R. B. (1976). A new goal programming formulation. *Omega*, 4(6), 731-732.

Garner, R. (1990). When children and adults do not use learning strategies: Toward a theory of settings. *Review of educational research*, 60(4), 517-529.

Hamachek, D. E. (1990). *Psychology in teaching, learning, and growth*. Allyn & Bacon.

Hannula, M. S., Maijala, H., & Pehkonen, E. (2004). Development of Understanding and Self-Confidence in Mathematics; Grades 5-8. *International Group for the Psychology of Mathematics Education*.

Jacobs, J. E., & Paris, S. G. (1987). Children's metacognition about reading: Issues in definition, measurement, and instruction. *Educational psychologist*, 22(3-4), 255-278.

Kazima, M., Pillay, V., & Adler, J. (2008). Mathematics for teaching: Observations from two case studies. *South African Journal of Education*, 28(2), 283-299.

Jagals, D., & Van der Walt, M. (2016). Enabling metacognitive skills for mathematics problem solving: A collective case study of metacognitive reflection and awareness. *African Journal of Research in Mathematics, Science and Technology Education*, 20(2), 154-164.

Jansen, J. D. (2011). *We need to talk.* Bookstorm and Macmillan.

Kilpatrick, J., Swafford, J., & Findell, B., 2001. *Adding it up. Helping Children Learn Mathematics.* National Academy Press.

Kember, D., Ho, A., & Hong, C. (2008). The importance of establishing relevance in motivating student learning. *Active learning in higher education*, 9(3), 249-263.

Koehler, M., & Mishra, P. (2009). What Is Technological Pedagogical Content Knowledge? Contemporary Issues in Technology and Teacher Education, 9(1), 60-70.

Kramarski, B., & Kohen, Z. (2017). Promoting preservice teachers' dual self-regulation roles as learners and as teachers: Effects of generic vs. specific prompts. *Metacognition and Learning*, 12(2), 157-191.

Lorch, R. F., Lorch, E. P., & Klusewitz, M. A. (1993). College students' conditional knowledge about reading. *Journal of educational psychology*, 85(2), 239-252.

Machaba, F. M. (2016). The concepts of area and perimeter: Insights and misconceptions of Grade 10 learners. *Pythagoras*, 37(1), 1-11.

MacMillan, J. H., & Schumacher, S. (2010). *Research in education: Evidence-based inquiry*. Pearson.

Mahlomaholo, S., 2013. On bricolage and the creation of sustainable postgraduate learning environments. TD: The Journal for Transdisciplinary Research in Southern Africa, 9(3), pp.379-392.

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, 108(6), 1017–1054.

Maree, K. (2010). First Steps in Research. Van Schaik.

Mwamwenda, T. S. (1995). Job satisfaction among secondary school teachers in Transkei. *South African Journal of Education*, 15(2), 84-87.

Myerson, J., Robertson, S., & Hale, S. (2007). Aging and intraindividual variability in performance: Analyses of response time distributions. *Journal of the Experimental Analysis of Behavior*, 88(3), 319-337.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. NCTM.

Ni Shuilleabhain, A., & Seery, A. (2018). Enacting curriculum reform through lesson study: A case study of mathematics teacher learning. *Professional development in education*, 44(2), 222-236.

Ngcoza, K.M. (2013). Science teachers' transformative and continuous professional development: A journey towards capacity-building and reflexive practice. http://hdl.handle.net/10962/d1008258.

Nyaumwe, L., Bappoo, R., Buzuzi, G., & Kasiyandima, O. (2004). *Students' Perceptions of Factors and Gender Differences that Influence their Achievement in 'O' Level Mathematics in Mashonaland Central Region*. University of Zimbabwe.

Ojose, B. 2011. Mathematics Literacy: Are We Able to Put the Mathematics We Learn into Everyday Use? *Journal of Mathematics Education*, 4(1), 89–100.

Ni Shuilleabhain, A., & Seery, A. (2018). Enacting curriculum reform through lesson study: A case study of mathematics teacher learning. *Professional development in education*, 44(2), 222-236.

Penprase, B. E. (2018). The fourth industrial revolution and higher education. In *Higher education in the era of the fourth industrial revolution* (pp. 207-229). Palgrave Macmillan, Singapore.

Rigelman, N. R. (2007). Fostering mathematical thinking and problem solving. *Teaching Children Mathematics*, 13(6), 308-314.

Reynders, A. (2014). *Obstacles that hamper learners from successfully translating mathematical word problems into number sentences* (Doctoral dissertation). University of the Free State.

Poulson, L. (2001). Paradigm lost? Subject knowledge, primary teachers and education policy. *British Journal of Educational Studies*, 49(1), 40-55.

Sanni, R. (2007). Teaching geometry in schools: An investigative rather than instructive process. *Pythagoras*, *0*(65), 39-44. https://doi.org/10.4102/pythagoras.v0i65.90

Schraw, G., & Moshman, D. (1995). Metacognitive theories. *Educational psychology review*, 7(4), 351-371.

Schuck, S. (1999). Teaching mathematics: A brightly wrapped but empty gift box. *Mathematics Education Research Journal*, *11*(2), 109-123.

Schwab, K. (2016). The Fourth Industrial Revolution: what it means, how to respond. In *World economic forum* (Vol. 14, No. 01, p. 2016). Crown Business.

Sepeng, J. P. (2010). *Grade 9 second-language learners in township schools: Issues of language and mathematics when solving word problems* (Unpublished Doctoral Dissertation). Nelson Mandela Metropolitan University.

Sheinuk, L. C. (2010). Intermediate phase mathematics Teachers reasoning about learners' mathematical thinking (Unpublished Master of Education Thesis). Wits School of Education.

Shepherd, D.L. (2012). The Impact of Subject Knowledge on Learner Performance in South Africa: *A within-pupil across-subject approach.* VrijeUniversiteit, Amsterdam, Netherlands. University of Stellenbosch, South Africa.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational researcher*, 15(2), 4-14.

Stein, M. K., Smith, M. S., Henningsen, M. A., & Silver, E. S. (2001). *Implementing standards-based mathematics instruction.* Teachers College Press.

Stacey, K., & Turner, R. (2015). *Assessing mathematical literacy.* Springer.

Suciati, Munadi, S., Sugiman, & Febriyanti, W. D. R. (2020). Design and validation of mathematical literacy instruments for assessment for learning in Indonesia. *European Journal of Educational Research*, 9(2), 865-875. https://doi.org/10.12973/eu-jer.9.2.865

Sweeting, K. 2011. *Early Years Teachers' Attitudes Towards Mathematics.* MEd Thesis. Queensland University of Technology.

Tachie, S. A. (2019). Meta-cognitive skills and strategies application: How this helps learners in mathematics problem-solving. E*URASIA Journal of Mathematics, Science and Technology Education*, 15(5), em1702.

Tachie, S. A., & Molepo, J. M. (2019). Exploring Teachers' Meta-Cognitive Skills in Mathematics Classes in Selected Rural Primary Schools in Eastern Cape, South Africa. *Africa Education Review*, 16(2), 143-161. https://doi.org/10.1080/18146627.2017.1384700

Tambara, C.T. (2015). *Unpacking teachers' pedagogical content knowledge and skills to develop learners' problem solving skills* (Doctoral dissertation). Stellenbosch University, South Africa

Tashakkori, A., & Teddlie, C. (2003). H*andbook of mixed methods in the social and behavioral sciences.* Sage.

Uberti, Z. H., Mastropieri, M. A., & Scruggs, T. E. (2004). Check it off: Individualizing a math algorithm for students with disabilities via self-monitoring checklists. *Intervention in School and Clinic*, 39(5), 269-275.

Visser, M., Juan, A., & Feza, N. (2015). Home and school resources as predictors of mathematics performance in South Africa. *South African Journal of Education*, 35(1).

Waters, H. S. E., & Schneider, W. E. (2010). *Metacognition, strategy use, and instruction*. Guilford Press.

Webb L. (2010). *Searching for Common Sounds: Developing Mathematical Reasoning through Dialogue* (Doctoral dissertation). Nelson Mandela Metropolitan University.

Wilson, J. (1998). *Meta-Cognition within Mathematics: A New and Practical Multi-Method Approach.* Proceedings of the 21st Annual Conference of the Mathematics Education Research Group of Australasia (MERGA), 693–700. Sydney: MERGA.

Wise, D.T. and Woodhouse, D.J., 2017. A cubical flat torus theorem and the bounded packing property. *Israel Journal of Mathematics*, *217*(1), 263-28.